TURBINE-BUCKET TEMPERATURES WITH PERIODIC VARIATION IN GAS TEMPERATURE

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The turbine-bucket temperature is determined for a sinusoidal variation in the gas temperature. It is demonstrated that the bucket temperature varies with the same frequency as the temperature of the gas, with the amplitude of bucket-temperature fluctuations being smaller than the amplitude of gas-temperature fluctuation.

In a variable regime the stator and rotor blades, the housings, disks, and similar components in the turbine of a gas-turbine engine exhibit temperatures which differ from the temperature of the stagnant gas flow. This is explained by the fact that a specific amount of time—dependent on the mass, surface, and heat capacity of the components, on the coefficient of heat transfer, and on the law governing the change in gas temperature with time—is required for the heating of these components. Experimentally confirmed calculations have demonstrated that the turbine-bucket temperature of a gas-turbine engine [GTE] is substantially lower (by 40–80°) than the gas temperature [1] at the end of acceleration, because of the thermal inertia of the buckets. From the standpoint of bucket [blade] strength it is frequently unnecessary, in acceleration regimes, to be concerned about great excesses of gas temperature [2] (when the thermal stresses are small, and when the over-all magnitude of the stresses does not exceed the elastic limit). It was assumed in the bucket-temperature calculations of [1, 2], as well as in [3], that the gas temperature increased instantaneously from its initial magnitude of t_{g_1} by a quantity Δt_g and then remained continuously constant and equal to $t_{g_2} = t_{g_1} + \Delta t_g$. These conditions are characteristic of stationary, marine, and particularly locomotive GTE in which, after acceleration, the gas temperature remains constant for a comparatively long period. However, many engines operate in a markedly variable regime in which the gas temperature, the rpm, and the engine power vary periodically. Among these we can include passenger-car and transport GTE, as well as aircraft engines when the aircraft is engaged in the execution of advanced flight maneuvers. There is a sufficient reduction in the strength and life of blades as a result of periodic variations in temperature, so that the determination of the service and over-all life of a gas-turbine engine requires familiarity with the manner in which the temperature of the engine's buckets changes in a variable regime. Under operational conditions the gas temperature can be measured and recorded on an oscillograph; however, the blade temperatures (particularly in the rotor) are not easily recorded under these conditions. The determination of the bucket temperatures for periodic variations in gas temperatures by means of calculation therefore makes it possible to achieve a result without resorting to complex measurements, at the same time also making it possible in the general case to analyze those factors which affect the thermal state of the blades.

The differential equation for the determination of temperatures in uncooled blades [buckets] as a function of time can be written as follows [1]:

$$A(t_g - t_b)\,d\tau = dt_b. \tag{1}$$

The temperature t_g for the stator blades correspnds to the stagnation temperature with respect to the absolute velocity of the inlet flow, while for the rotor blades it corresponds to the stagnation temperatures with respect to the relative velocity. We will examine the average blade temperature over the entire cross section. If necessary, we will determine the temperature of the trailing edge in approximate terms (without consideration of heat conduction); for this the coefficient A should be calculated for the F_b/G_b ratio corresponding to the surface and the weight of the blade in the region of the trailing edge. This ratio is proportional to the perimeter of the profile section under consideration divided by its lateral cross section.

The general solution for the linear equation (1) has the form

$$t_b = \exp[-A\tau]\,\{At_g \exp[A\tau]\,d\tau + C\}. \tag{2}$$

If we know the law governing the change in the gas temperature and in the quantity A with time, the blade temperature can always be determined in quadratures. If the integral in Eq. (2) is not taken in final form, the problem is more simply resolved by an approximate solution of Eq. (1)—by the method of broken Euler lines, by means of power series, etc.

With a periodic change in temperature, the quantity t_g in (2) can be represented by a Fourier series. The quantity A is a function of time in view of the fact that the heat-transfer coefficient is not constant. If A is variable, the integral in (2) is not expressed in elementary functions, thus complicating the solution of the problem and the analysis of the derived result. In connection with the fact that the heat-transfer coefficient does not generally vary markedly, in the determination of the temperature of the blades flushed by a stream with a periodically varying temperature, we can assume that the value of α_g is constant and equal to the time-averaged coefficient of heat transfer from the gas to the blades. The quantity A is constant in this case and if the gas temperature with respect to time is expressed by a Fourier series, the integrals in (2) are brought to the form $\int \exp x \sin x\,dx$ or $\int \exp x \cos x\,dx$, and these are easily taken.

Let us solve the problem in which the gas temperature varies with time according to the sinusoidal law

$$t_g = \frac{t_{g1} + t_{g2}}{2} + \frac{t_{g2} - t_{g1}}{2} \sin\left(\omega\tau - \frac{\pi}{2}\right). \quad (3)$$

It follows from Eq. (3) that the gas temperature at the initial instant ($\tau = 0$) is equal to its initial value t_{g1}. Although the case of the sinusoidal variation in temperature is the simplest of the numerous laws governing the periodic variation in temperature, it is, nevertheless, sufficiently general in many cases and, moreover, a number of general quantitative relationships can be established on the basis of a sinusoidal law governing the change in t_g.

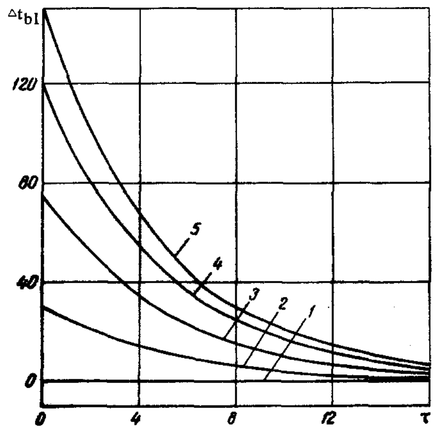

Fig. 1. Δt_{bI} (°C) versus time (sec) at $t_{g_2} - t_{g_1} = 300°$ C; $A = 0.2$ sec^{-1} and various relations ω/A: 1) $\omega/A = 0$; 2) 0.5; 3) 1; 4) 2; 5) ∞.

Substitution of (3) into (2) makes it possible to derive a general solution for (1) in the case of a sinusoidal variation in the gas temperature:

$$t_b = \frac{t_{g1} + t_{g2}}{2} + \frac{A}{A^2 + \omega^2} \frac{t_{g2} - t_{g1}}{2} \times$$
$$\times \left[A \sin\left(\omega\tau - \frac{\pi}{2}\right) - \omega \cos\left(\omega\tau - \frac{\pi}{2}\right)\right] +$$
$$+ C \exp[-A\tau]. \quad (4)$$

We find the arbitrary constant C from the condition that when $\tau = 0$ the blade temperature is equal to t_{b_0}. From (4) for $\tau = 0$ it then follows that

$$C = t_{b_0} - \frac{t_{g1} + t_{g2}}{2} + \frac{A^2}{A^2 + \omega^2} \frac{t_{g2} - t_{g1}}{2}.$$

Having substituted this quantity into (4) and having assumed that $t_{b_0} = t_{g1}$ (i.e., the gas temperature prior to the onset of the nonsteady regime and the blade temperature coincide), after transformations we derive an equation for the determination of the blade temperature for a sinusoidal variation in the gas temperature:

$$t_b = t_{g_{av}} - \frac{t_{g2} - t_{g1}}{2\exp A\tau}\left(1 - \frac{1}{1 + \frac{\omega^2}{A^2}}\right) +$$
$$+ \frac{t_{g2} - t_{g1}}{2\sqrt{1 + \frac{\omega^2}{A^2}}} \times$$
$$\times \sin\left(\omega\tau - \frac{\pi}{2} - \text{arctg}\,\frac{\omega}{A}\right), \quad (5)$$

where $t_{g_{av}} = t_{g1} + t_{g2}/2$ is the average gas temperature.

The phase angle $\varphi = \text{arctg}(\omega/A)$ shows here the extent to which there has been a phase shift in the sinusoid showing the change in the gas temperature.

It follows from Eq. (5) that the blade temperature for a simple harmonic variation in the gas temperature can be demonstrated to consist of three terms: a constant equal to the average gas temperature, an exponential term, and one that is sinusoidal, i.e.,

$$t_b = t_{g_{av}} - \Delta t_{bI} + \Delta t_{bII}, \quad (6)$$

where

$$\Delta t_{bI} = \frac{t_{g2} - t_{g1}}{2\exp A\tau}\left(1 - \frac{1}{1 + \frac{\omega^2}{A^2}}\right);$$

$$\Delta t_{bII} = \frac{t_{g2} - t_{g1}}{2\sqrt{1 + \frac{\omega^2}{A^2}}} \sin\left(\omega\tau - \frac{\pi}{2} - \text{arctg}\,\frac{\omega}{A}\right).$$

In accordance with Eq. (6), the curve showing the change in the blade temperature can be divided into two segments—an initial segment and a primary segment. The exponential term in the initial segment has a marked effect on the blade temperature and the curve $t_b = f(\tau)$ in this segment will not be periodic. The relationship between Δt_{bI} and time for the various ω/A ratios is shown in Fig. 1. We see that the quantity Δt_{bI} increases with an increase in ω/A, varying from zero when $\omega/A = 0$ to $(t_{g_2} - t_{g_1})/2$ when $\omega/A = \infty$ and $\tau = 0$. The value of Δt_{bI} rapidly diminishes with an increase in τ and approaches very close to zero in value for virtually all actual magnitudes of ω/A when $\tau = 10-15$ sec. If we assume that the effect of the exponential term can be neglected when $\Delta t_{bI} < 0.01\,\Delta t_{bII}$, the blade temperature as a function of time consequently becomes periodic from this instant on. For quantities $A = 0.2-0.4$ sec^{-1}, usual for blades, the

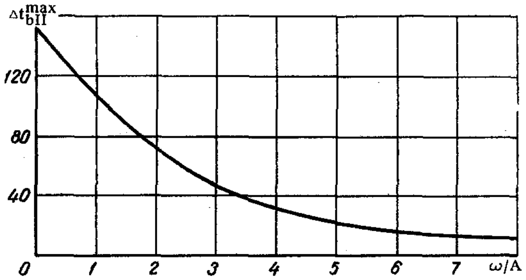

Fig. 2. Amplitude of blade-temperature oscillations in the main region versus ω/A with sinusoidal variation of gas temperature and $t_{g_2} - t_{g_1} = 300°$ C.

curve $t_b = f(\tau)$ becomes periodic within 10–20 sec after the onset of the nonsteady regime. In the solution of the majority of practical cases for a sufficiently prolonged periodic variation in gas temperature it therefore becomes possible to assume that $\Delta t_{bI} = 0$ and the calculations can be carried out according to the formula

$$t_b = t_{g_{av}} + \Delta t_{bII} = t_{g_{av}} + \frac{t_{g2} - t_{g1}}{2\sqrt{1+\frac{\omega^2}{A^2}}} \times$$

$$\times \sin\left(\omega\tau - \frac{\pi}{2} - \operatorname{arctg}\frac{\omega}{A}\right). \qquad (7)$$

We will analyze the derived equations for the determination of blade temperatures in the case of a sinusoidal time-variation in the gas temperature. As we can see from Eq. (5), the nature of the variation in the blade temperature is a strong function of the dimensionless ratio ω/A. We can examine two extreme cases:

a) If $\omega/A = 0$, which may be the case when $A = \infty$ ($\omega = 0$ is the case in which there are no fluctuations and is thus of no interest), i. e., with infinitely small thermal inertia for the blades, their temperature will be expressed by the following equation:

$$t_b = t_{g_{av}} + \frac{t_{g2} - t_{g1}}{2}\sin\left(\omega\tau - \frac{\pi}{2}\right).$$

It follows from a comparison of this expression relative to (3) that the blade temperature coincides with that of the gas. The amplitude of the fluctuations in the blade temperature will be at its maximum in this case.

b) If $\omega/A = \infty$ which corresponds to $A = 0$, the blade temperature will be equal to

$$t_b = t_{g_{av}} = \frac{t_{g1} + t_{g2}}{2},$$

i. e., it will be constant and coincide with the average gas temperature, while the amplitude for the fluctuation in the blade temperature is equal to zero. It is not difficult to see that this case corresponds to an infinitely great thermal inertia for the blade.

All of the practically realizable laws governing the variation in t_b lie between these two extreme cases. The amplitude of the fluctuation in blade temperature (on the primary segment of the variation) will be defined by the expression

$$\Delta t_{bII}^{max} = (t_{g2} - t_{g1}) \Big/ 2\sqrt{1 + \frac{\omega^2}{A^2}},$$

i. e., the quantity Δt_{bII}^{max} will be smaller, the larger the ratio ω/A (an analogous result can be derived by subjecting Eq. (1) to a Laplace transformation) (Fig. 2), in which case the two above-considered extreme cases follow directly from the above-cited equation.

For a clear presentation of the nature of the change in blade temperature with time in the case of a sinusoidal variation in gas temperature, we carried out the calculation according to formula (5) with the following initial conditions: $t_{g_1} = 600°$ C, $t_{g_2} = 900°$ C, $\omega = 0.2\pi = 0.628$ sec^{-1} and $A = 0.4$ sec^{-1}. The change in the gas temperature and in the blade temperature with time is shown in Fig. 3. We see from the cited curves that the maximum blade temperature does not exceed 830° C, i. e., lower by 70° than the maximum gas temperature. The amplitude for the fluctuation in the blade temperature on the primary segment is $\Delta t_{bII}^{max} = 81°$ C which is considerably lower than the amplitude of the gas temperature which is equal to 150° C. The primary segment of fluctuations in blade temperature begins within 11 sec and the fluctuations then become harmonic and their amplitude assumes a constant value. The maximum blade temperature on this segment is shifted relative to the maximum of the temperature curve by the angle of the phase shift which is equal to arctg ω/A and in the case under consideration this shift (with respect to time) amounts to 1.5 sec. The thermal inertia of the blades thus leads to a reduction in their maximum temperature and in the amplitude of the fluctuation relative to the amplitude

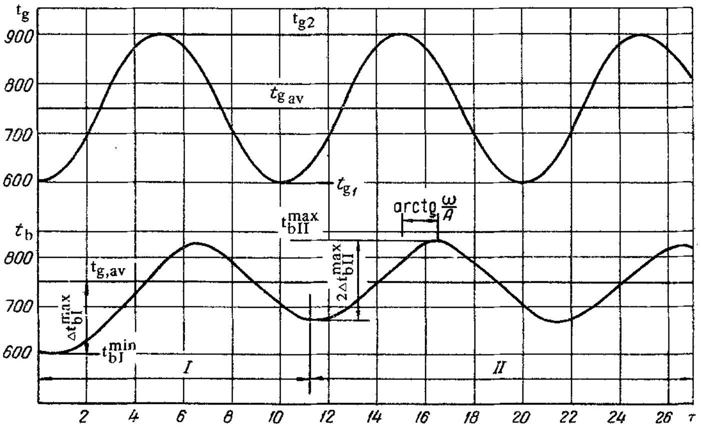

Fig. 3. Blade temperature with sinusoidal law of temperature variation for the gas and under initial conditions: $A = 0.4$ sec^{-1}; $\omega = 0.2\pi$ sec^{-1}; $T = 10$ sec; $\Delta t_{bII}^{max} = 81°$ C; I) initial region; II) main region.

of the fluctuation in gas temperature, which has a favorable effect on the dynamic strength of the blades.

In conclusion we will demonstrate that the time constant A in Eq. (1) may be assumed to be constant in the majority of cases. This quantity is proportional to the coefficient of the heat transfer from the gas to the blade, i.e.,

$$A = \mathrm{const}\,\alpha_g.$$

The coefficient of heat transfer from the gas to the blade is equal to [1]

$$\alpha_g = \frac{\mathrm{Re}^{0.545}\lambda_g}{b} = \mathrm{const}\frac{G_g^{0.545}\lambda_g}{\mu_g^{0.545}}.$$

If the coefficients of heat conduction and dynamic viscosity as known functions of temperature (interpolational formulas) are substituted into this last equation, with consideration of the fact that with constant pressure the gas flow rate is inversely proportional to the square root of the temperature, we will have

$$\alpha_g = \mathrm{const}\,T_g^{0.1}$$

and, consequently,

$$A = \mathrm{const}\;T_g^{0.1}. \tag{8}$$

The amplitude for the fluctuation in the gas temperature generally does not exceed 150–200°, i.e., the gas temperature varies by no more than 15–25% in comparison with the quantity $T_{g_{av}}$. As follows from formula (8), the quantity A therefore changes by no more than 1.5–2.5%, and it may be regarded as constant. In the event that there is a change in the gas pressure, the quantity A varies more markedly, but in this case its variation can be neglected in most cases.

NOTATION

A is a constant; α_g is the heat-transfer coefficient from gas to blade; F_b, G_b, and c_b are the surface, weight, and heat capacity of blade; ω is the frequency; T is the period; τ is the time; t_b is the temperature of the blades; T_g and t_g are the gas temperatures; G_g is the gas flow rate; b is the blade chord; μ_g is the dynamic viscosity coefficient of the gas; λ_g is the thermal conductivity of the gas.

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